Rewarding Learning

General Certificate of Secondary Education
2023

Centre Number

Candidate Number

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

MV18

[GFM11]

FRIDAY 2 JUNE, MORNING

Time

2 hours, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Complete in black ink only.
All working **must** be clearly shown in the spaces provided.
Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fourteen** questions.

Information for Candidates

The total mark for this paper is 100.
Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 2.

Formula Sheet

Pure Mathematics

Quadratic equations: If $ax^2 + bx + c = 0 \qquad (a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c \qquad (n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$(ad - bc \neq 0)$

1 Find $\frac{dy}{dx}$ if $y = \frac{2}{3}x^6 + \frac{1}{2x^4}$ [2 marks]

Answer _____

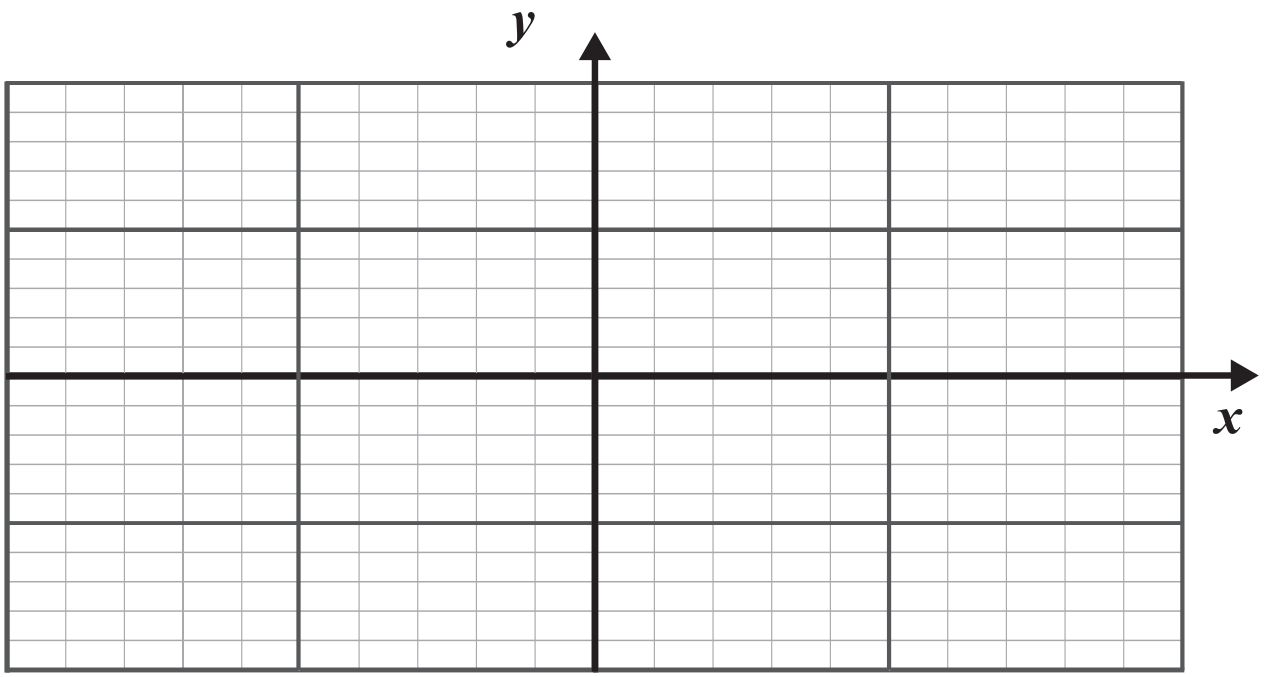
2 Express y in terms of x if

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 4$$

given that $y = 7$ when $x = 2$ [5 marks]

Answer $y =$ _____

3 **(a)** Sketch the graph of $y = \cos x$ for $-180° \leqslant x \leqslant 180°$ on the axes below. [2 marks]



(b) (i) Solve the equation [3 marks]

$3 \cos x = -2$

for $-180° \leqslant x \leqslant 180°$

Answers _____

(ii) Hence solve the equation [3 marks]

$$3 \cos\left(\frac{\theta}{2} - 10°\right) = -2$$

for $-360° \leqslant \theta \leqslant 360°$

Answers _____

4 A function f(x) is defined by

f(x) = $x^2 - 8x + 24$

(i) Use the method of **completing the square** to rewrite f(x) in the form

$(x - a)^2 + b$

where a and b are constants. [2 marks]

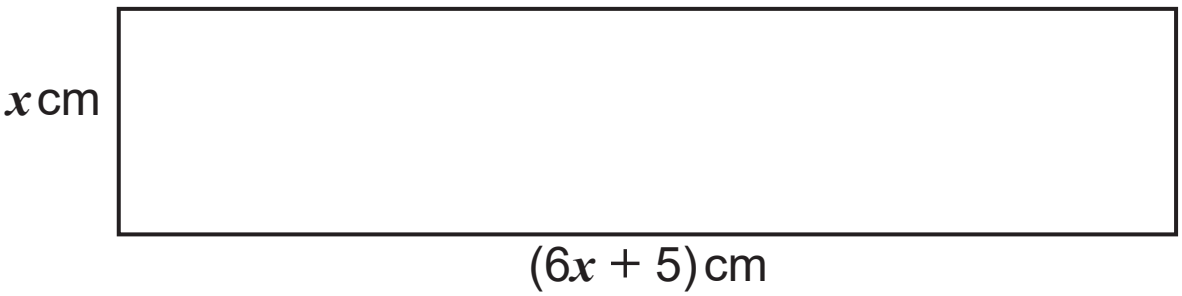
Answer _____

(ii) Hence write down the coordinates of the minimum turning point of the curve [2 marks]

$y = x^2 - 8x + 24$

Answer _____

5 A rectangle has width x cm and length $(6x + 5)$ cm, as shown below.



Find the range of values of x for which the area of the rectangle is less than $6\,\text{cm}^2$ [6 marks]

You **must** show clearly each stage of your solution.

Answer _____

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(Questions continue overleaf)

6 Matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 5 & -2 \\ 4 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{C} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

(i) Solve the matrix equation $\mathbf{AX} = \mathbf{C}$ [4 marks]

Answer _____

(ii) Explain why it is impossible to solve the equation $\mathbf{BX} = \mathbf{C}$ [1 mark]

Answer _____

_____

7 **(a)** If $\log_7(3) = a$ and $\log_7(2) = b$

express $\log_7(10.5)$ in terms of a and b. [2 marks]

Answer _____

(b) Solve the equation [4 marks]

$$9^{\left(\frac{2}{5}x - 2\right)} = 4$$

Answer _____

8 Matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \quad \mathbf{B} = [3 \ \ -1] \text{ and } \quad \mathbf{C} = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$$

Calculate [2 marks]

(i) $\mathbf{AB}$

Answer _____

(ii) $\mathbf{C^2 - AB}$ [3 marks]

Answer _____

9 Simplify **fully** the following algebraic expression
[4 marks]

$$\left(\frac{x}{2} - \frac{2}{x}\right) \div \frac{(x+2)}{6x}$$

Answer _____

10 A curve is defined by the equation $y = 2x^3 - 3x^2 + x + 5$

Find the equation of the **normal** to the curve at the point P (–1, –1). [7 marks]

Answer _____

11 Jack, Jill and Jared individually deposited money in each of three banks, A, B and C.

Bank A pays x% interest each year.

Bank B pays y% interest each year.

Bank C pays z% interest each year.

Jack deposited £1500, £3500 and £4500 in banks A, B and C respectively, and received £305 overall interest in a year.

(i) Show that x, y and z satisfy the equation [1 mark]

$$3x + 7y + 9z = 61$$

Jill deposited £1000, £2000 and £6000 in banks A, B and C respectively, and received £330 overall interest in a year.

(ii) Show that x, y and z also satisfy the equation
[1 mark]

$$x + 2y + 6z = 33$$

Jared deposited £2000, £3000 and £5000 in banks A, B and C respectively, and received £320 overall interest in a year.

(iii) Show that x, y and z also satisfy the equation
[1 mark]

$$2x + 3y + 5z = 32$$

(iv) Solve the equations below to find the percentage interest per year paid by each of the banks, showing clearly each stage of your solution. [8 marks]

$$3x + 7y + 9z = 61$$

$$x + 2y + 6z = 33$$

$$2x + 3y + 5z = 32$$

Answer Bank A _____%

Bank B _____%

Bank C _____%

12 A curve is defined by the equation $y = (2x - 1)(x - 2)^2$

(i) Write down the **coordinates** of the points where the curve meets the x-axis. [2 marks]

Answer _____

(ii) Write down the **coordinates** of the point where the curve meets the y-axis. [1 mark]

Answer _____

(iii) Expand fully the expression $(2x - 1)(x - 2)^2$ [3 marks]

Answer _____

(iv) Find the coordinates of the turning points of the curve.
[6 marks]

Answer _____

(v) Using calculus, identify each turning point as either a maximum or a minimum point. [2 marks]

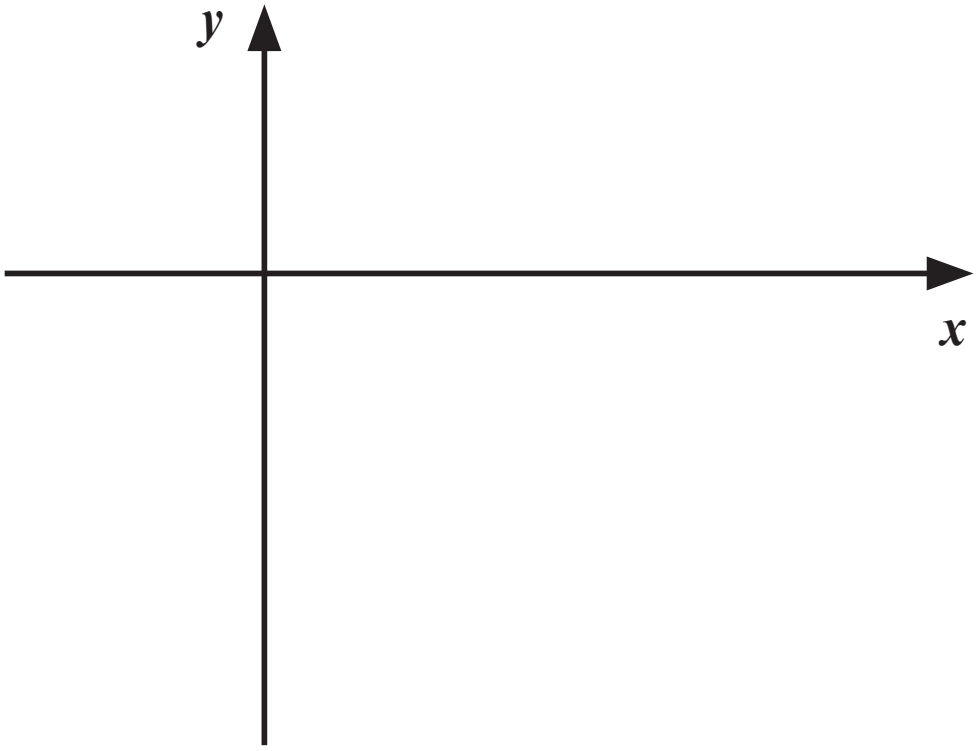
You **must** show working to justify your answer.

Answer _____

(vi) **Hence**, using your answers from parts **(i)** to **(v)**, sketch the curve of

$y = (2x - 1)(x - 2)^2$

on the axes below. [3 marks]



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(Questions continue overleaf)

13 John took out an investment 20 years ago. Over the next 20 years he recorded the value of his investment at various times.

The table below shows the value of his investment, V, in pounds, after T years.

Years T	**Value** V **(£)**		
3	673.14		
5	715.70		
8	757.22		
14	809.18		
20	845.23		

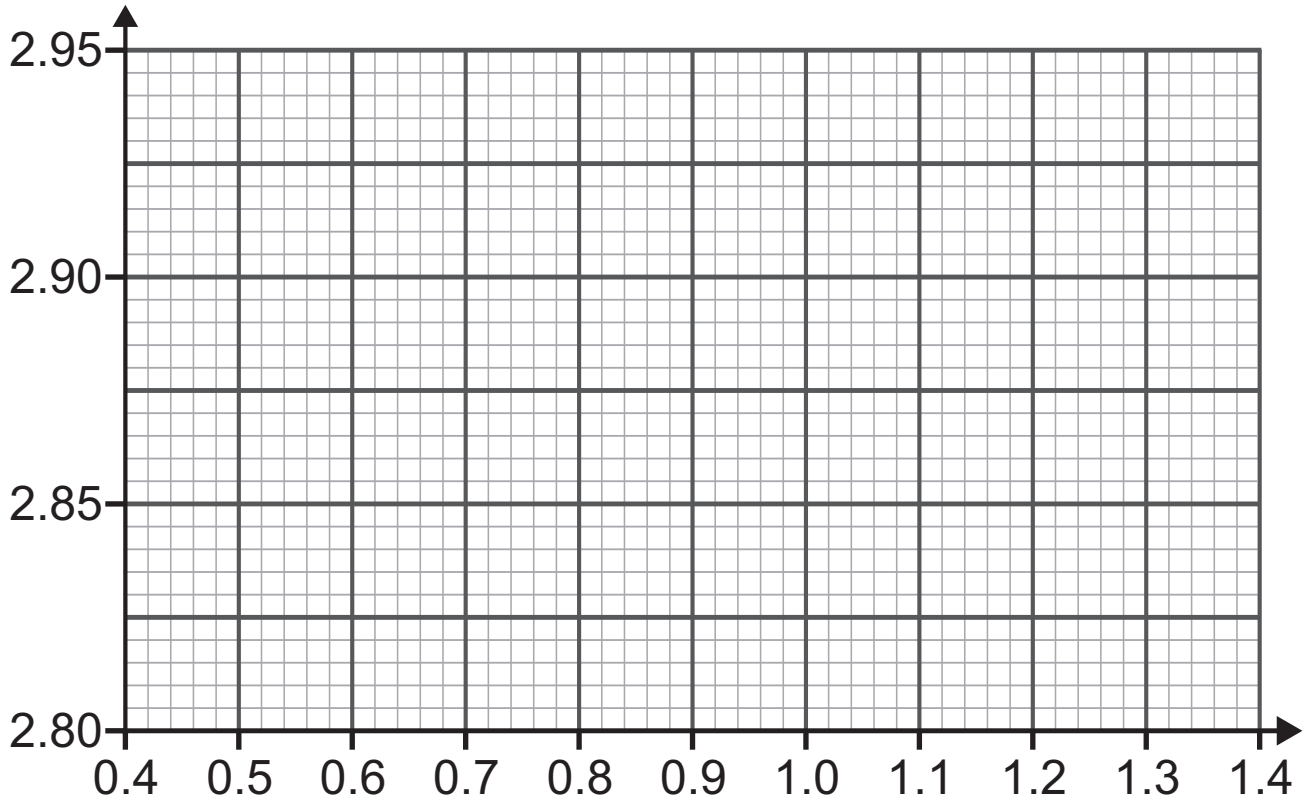
It is believed that a relationship of the form

$$V = aT^n$$

exists between years 3 and 20, where a and n are constants.

(i) Verify that a relationship of the form $V = aT^n$ exists by drawing a suitable straight line graph on the grid opposite. [6 marks]

Show clearly the values used, correct to 3 decimal places, in the table above.


2.95
2.90
2.85
2.80
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4

(ii) **Hence**, find the values of a and n, giving a correct to the nearest integer and n correct to 2 decimal places.
[4 marks]

Answer $a =$ _____ , $n =$ _____

(iii) Use the formula $V = aT^n$, with your values for a and n, to calculate after how many years the value of the investment was **£**830 [2 marks]

Give your answer correct to 1 decimal place.

You **must** show your working,

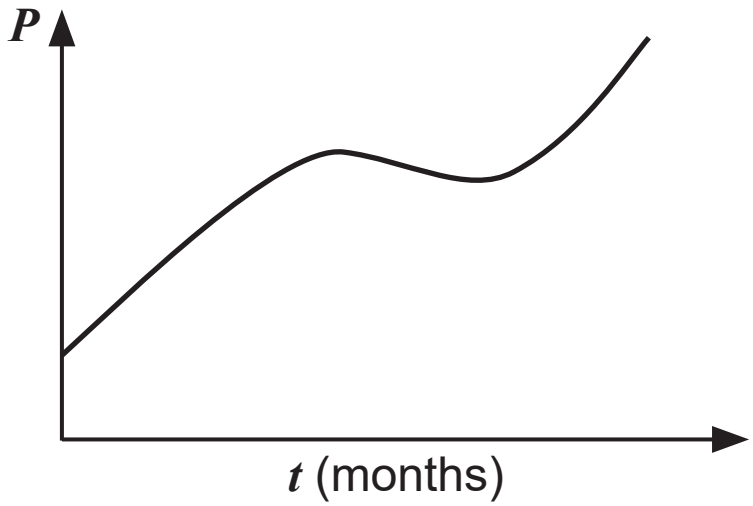
Answer _____ years

14 On a remote island the population P of lizards is modelled by the function

$$P = t^3 - 12t^2 + 45t + 10$$

where t is the time in months since the counting of lizards began.

A sketch of the population of lizards against time in months is shown below.



(i) Write down the initial population of lizards when counting began. [1 mark]

Answer _____

(ii) Using calculus, find the time, in months, after which the population **started to fall**. [5 marks]

Answer _____

(iii) What was the minimum number of lizards after the population started to fall, before it began to rise again? [2 marks]

Answer _____

This is the end of the question paper

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
Total Marks	

Examiner Number